LITERATURE CITED

1. G. I. Kelbaliev, Teor. Osn. Khim. Tekhnol., 16, No. 1, 38-43 (1982).
2. G. I. Kelbaliev, Teor. Osn. Khim. Tekhnol., 17, No. 3, 330-336 (1983).
3. A. A. Zhukauskas, Convective Transfer in Heat Exchangers [in Russian], Moscow (1982).
4. A. D. Al'tshul', Hydrodynamic Resistances [in Russian], Moscow (1970).
5. G. I. Kelbaliev, Inzh.-Fiz. Zh., 46, No. 6, 1023-1024 (1984).
6. G. I. Kelbaliev, Teor. Osn. Khim. Tekhnol., 19, No. 5, 616-621 (1985).

DETERMINATION OF THE THERMOPHYSICAL CHARACTERISTICS BY THE SELF-OSCILLATION METHOD

V. P. Alekseev, S. E. Birkgan, Yu. N. Burtsev,
A. S. Rudyi, and S. N. Shekhtman

UDC 536.2

A method is proposed for measuring the thermophysical characteristics by means of the self-oscillation frequency and the gain coefficient of the automatic regulation system containing the specimen under investigation.

The phenomenon of exciting self-oscillations in automatic regulation systems is well known [1, 2]. For a regulator of low inertia the self-oscillation frequency depends only on the physical characteristics of the object of regulation. If material with unknown thermophysical characteristics is taken as such an object, and a temperature stabilizer as regulator, then stable, almost sinusoidal, temperature oscillations can be obtained in the specimen under investigation. The frequency of these oscillations permits an assessment of the thermal diffusivity of the material. The heat conduction equation with nonlinear boundary conditions will be the mathematical model of this system. Similar equations with a weak nonlinearity are investigated by asymptotic methods. Thus, an algorithm to compute the self-oscillation in nonlinear parabolic systems with a small parameter [3] used in this paper was developed and given a foundation comparatively recently. Asymptotic methods are based on the fact that the desired periodic solution is bifurcated from the equilibrium state as the small parameter increases. Let us note that the bifurcation of periodic solutions can occur only in the case of a nonlinearity of a definite kind (soft excitation mode), the amplitude of the self-oscillations here diminishes together with the parameter. In the opposite case the amplitude of the periodic solutions does not decrease with the diminution of the parameter (hard excitation mode) and asymptotic methods are unsuitable. Although the self-oscillation frequency of a system with such kind of nonlinearity indeed contains information about the thermophysical characteristics of the material, it is not possible to extract it without relying on numerical methods. In other words, the presence of self-oscillations without making the kind of nonlinearity specific cannot be used to determine the thermal diffusivity. Unfortunately, these well-known facts are not always taken into account [4]. Without delving into an analysis of the problem, the authors of the mentioned paper try to obtain a relationship between the frequency and the thermal diffusivity by assuming that the phase shift of the temperature oscillations in the specimen equals π and is 2π in combination with the phase shift of the inverting amplifier signal. Overlooked here is that a phase shift also exists between the power liberated in the heater and its temperature by virtue of the integrating properties of the specimen. Then the total phase shift in the system exceeds 2π which contradicts the self-oscillation condition. This and other examples indicate the necessity of a complete analysis of such systems.

Let us turn to a description of one of the possible methods of realizing the self-oscillation method, the construction of its mathematical model, and also the derivation of

N. S. Kurnakov Institute of General and Inorganic Chemistry, Academy of Sciences of the USSR, Moscow. Yaroslavsk State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 2, pp. 255-260, February, 1987. Original article submitted October 28, 1985.

 0022-0841/87/5202-0190$12.00

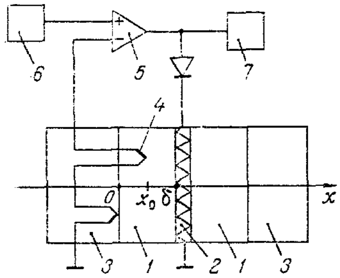

Fig. 1. Self-oscillating system with plane specimens.

formulas to calculate the heat conduction and thermal diffusivity. To organize the automatic regulation system, a specimen of material to be studied is set in contact with a heater and a heat sensor, which is connected to a temperature regulator, for instance, a differential amplifier. In this case the signal from the heat sensor is delivered to the amplifier inverting input while the output voltage goes to the heater. In a system obtained in such a manner, self-oscillations can be excited under definite conditions. The self-oscillation frequency depends only on the geometry and thermal diffusivity of the specimen, which permits a calculation of the thermal diffusivity coefficient in each specific case.

As an illustration, we consider the case when the specimens are in the shape of plates (sketch). Let two specimens 1 being investigated, separated by a plane low-inertia heater 2, be placed between two thermostats 3. A thermocouple 4 is inserted within one of the specimens and its signals goes to the inverting input of the differential power amplifier 5 with a regulatable gain coefficient. A reference voltage from the source 6 is delivered to the second input. A voltage proportional to the difference between the input signals goes from the output of the amplifier 5 to the heater 2. The recorder 7 measures the specimen temperature and the self-oscillation frequency.

The specimen, thermocouple, amplifier, and heater comprise an automatic regulation system described by the equations

$$\dot{T} = aT'', \; T_{x=0} = 0,$$
$$T'_{x=\delta} = \frac{K}{2\lambda SR}[u_0 - \alpha T(x_0, t)]^2 \sigma [u_0 - \alpha T(x_0, t)]. \tag{1}$$

The system (1) has a stationary solution of the form

$$T_{\text{st}}(x) = \frac{u_0}{\alpha x_0}(D \pm \sqrt{D^2 - 1})\, x, \tag{2}$$

where

$$D = 1 + \frac{\lambda SR}{K^2 \alpha u_0 x_0}.$$

It is evident that the Heaviside function in the second boundary condition of the system (1) should be positive in the stationary solution. The following condition hence results

$$u_0 [1 - (D \pm \sqrt{D^2 - 1})] > 0,$$

from which it follows that the sign in front of the radical should be negative. We linearize the nonlinear boundary-value problem (1) by the stationary solution (2). Consequently, we obtain a system of equations

$$\dot{T}(x, t) = aT''(x, t), \quad T(x, t)_{x=0} = 0,$$
$$T(x, t)_{x=\delta} = -\frac{K^2 \alpha}{\lambda SR}[u_0 - \alpha T_{\text{st}}(x_0)]\, T(x_0, t), \tag{3}$$

from whose form it follows that it has a periodic solution for certain gain coefficients

$$T(x,\ t) = V(x)\exp i\omega t. \tag{4}$$

Substituting the function (4) into the system (3), we obtain a boundary value problem to determine V(x):

$$V''(x) = \frac{i\omega}{a} V(x), \quad V(x)_{x=0} = 0,$$
$$V'(x)_{x=\delta} = -\frac{K^2\alpha}{\lambda SR} [u_0 - \alpha T_{st}(x_0)]\, V(x_0). \tag{5}$$

The solution of this problem has the form

$$V(x) = C\,\mathrm{sh}\,[k(1+i)\,x], \tag{6}$$

where the wave number is $k = \sqrt{\frac{\omega}{2a}}$. We find the spectrum of the wave numbers k by solving an eigennumber problem of the operator. Substituting (6) into the second boundary condition of (5), separating into real and imaginary parts, and introducing the notation $\nu = k\delta$, $n = x_0\delta^{-1}$ and $B = -\frac{K^2\alpha}{\lambda SR}[u_0 - \alpha T_{st}(x_0)]$, we obtain

$$\frac{\nu}{\delta}(\mathrm{ch}\,\nu\cos\nu - \mathrm{sh}\,\nu\sin\nu) = B\,\mathrm{sh}\,n\nu\cos n\nu,$$
$$\frac{\nu}{\delta}(\mathrm{ch}\,\nu\cos\nu + \mathrm{sh}\,\nu\sin\nu) = B\,\mathrm{ch}\,n\nu\sin n\nu. \tag{7}$$

Eliminating B in the expressions (7), we find a condition for the wave numbers k

$$\frac{\mathrm{ch}\,\nu\cos\nu - \mathrm{sh}\,\nu\sin\nu}{\mathrm{sh}\,n\nu\cos n\nu} = \frac{\mathrm{ch}\,\nu\cos\nu + \mathrm{sh}\,\nu\sin\nu}{\mathrm{ch}\,n\nu\sin n\nu}. \tag{8}$$

This equation has an infinite number of roots ν_j, j = 0, 1, ..., to each of which its value of the gain coefficient, determined from the system (7), corresponds. The least value of the gain for which the system (7) has a solution will be denoted by K_0. It follows from a linear analysis performed and from the results of [3] that the nonlinear system (1) has a periodic solution for $K = K_0$ that bifurcates from the stationary solution. The frequency of the periodic solution can here be determined with any degree of accuracy from the formula

$$\omega(\varepsilon) = \omega_0 + \sum_{j=1}^{n} \varepsilon^j \delta_j + o(\varepsilon^n), \tag{9}$$

where ω_0 is the self-oscillation frequency of the linear system (3) for $K = K_0$; $\varepsilon = (K - K_0)\cdot K_0^{-1}$; and δ_j are constants determined by the method in [3].

An investigation performed on the mathematical model permitted development of a measurement method and obtaining formulas to determine the heat conduction and thermal diffusivity coefficients. The measurements were executed according to the following scheme. By using the thermostats 3 (see sketch) an initial temperature was set up. Then the heater was switched in and by increasing the gain coefficient of the differential amplifier 5 excitation of temperature oscillations in the specimen was achieved. As the gain coefficient increased, the gradient of the stationary temperature evidently rises but, as follows from (2), does not exceed the quantity $u_0(\alpha x_0)^{-1}$ for any values of K. Consequently, the reference voltage is selected so that the temperature drop in the specimen would be 1.5-2°C for $K = K_0$. After the occurrence of stable temperature oscillations, the gain coefficient is diminished so that the amplitude of the oscillations would be minimal but the amplifier drift and noise would not here exert substantial influence on the mode of the oscillations, which should be almost sinusoidal. Then by using the recorder 7 the self-oscillation frequency is measured, whereupon the number of oscillations is computed per time interval whose duration was determined by accuracy considerations. Since the thermocouple was placed at the center of the specimen in our experiments (i.e., $n = 0.5$, $\nu_0 = 4{,}694{,}105$) and the dependence $\omega(\varepsilon)$ turned out to be quite weak, the formula $a = 0.02269\ \omega(\varepsilon)\delta^2$ was used to compute the thermal diffusivity. The heat-conduction coefficient was determined from the relationship

$$B_0 = \frac{K_0\alpha}{\lambda SR}\,[\alpha T_{st}\,(x_0) - u_0]. \tag{10}$$

It follows from (7) that

$$B_0 = -69{,}38\delta^{-1}. \tag{11}$$

Substituting (11) and (2) into (10) yields

$$\lambda = 1{,}571\cdot 10^{-3}\,\frac{\alpha u_0 x_0}{SR}K_0, \text{ or}$$

$$\lambda = A u_0 \delta K_0^2, \tag{12}$$

where A is the device constant determined experimentally.

To verify the method, heat-conduction and thermal diffusivity measurements were performed on polymethylmetacrylate and polytetrafluorethylene according to the scheme described above. The measurements were executed on specimens in the shape of 20-mm-diameter discs. One specimen was continuous and 4 mm thick, while a second consisted of two discs 2 mm thick each. Between the halves of the composite specimen was the junction of a differential thermocouple from foil also in the shape of a 20-mm disc 0.1 mm thick. The second junction was on the thermostatted surface of the specimen. The problem of the experiment was to verify the adequacy of a real system of the problem considered above. Special attention was paid to the self-oscillation excitation mode, the dependence of their frequency on the gain coefficient, the stability of the oscillations, and the influence of fluctuations, the accuracy of the measurement was not given decisive value here.

Let us estimate the error in measuring the thermal diffusivity coefficient. Logarithmic differentiation of the expression

$$a = \frac{[\omega(\varepsilon) - \beta(\varepsilon)]\,\delta^2}{2\nu_0^2},$$

where

$$\beta(\varepsilon) = \sum_{j=1}^{n} \varepsilon^j \delta_j + o(\varepsilon^n)$$

is the linearization error, yields the relative error in measuring the thermal diffusivity

$$\frac{\Delta a}{a} = \frac{\Delta\omega(\varepsilon)}{\omega(\varepsilon) - \beta(\varepsilon)} + \frac{\Delta\beta(\varepsilon)}{\omega(\varepsilon) - \beta(\varepsilon)} + \frac{2\Delta\delta}{\delta} + \frac{2\Delta\nu_0}{\nu_0}. \tag{13}$$

Since no noticeable change in frequency was detected experimentally as ε increased from zero until the appearance of nonlinear distortions for this kind of nonlinearity, the expression (13) can be represented in the form

$$\frac{\Delta a}{a} = \frac{\Delta\omega}{\omega} + \frac{2\Delta\delta}{\delta} + \frac{2\Delta\nu_0}{\nu_0}. \tag{14}$$

The circular self-oscillation frequency was determined with 0.5% error by a simple measurement of the time of an integral number of oscillations. The error in measuring the specimen thickness was 0.1% and the greatest contribution was a fraction of the last component in (13). Here $\Delta\nu_0$ is the indeterminacy of the root of the characteristic equation (8) due to the error in clamping the thermocouple Δx_0. The relation between them, determined from the condition that the total differentials of both sides of (8) are equal, has the form:

$$\frac{\Delta\nu_0}{\nu_0} = 2{,}072\frac{\Delta x_0}{\delta}.$$

In our experiments, the quantity Δx_0 was $1.5\cdot10^{-2}$ mm, $\Delta\nu_0/\nu_0$ was 0.8%, and the total error should not exceed 1.5%. However, the values of the thermal diffusivity obtained as a result of measuring standard specimens were 7% below specifications. The discrepancy between the computed and experimental errors is explained mainly by the nonuniformity of the heat flux in the specimens. Indeed, the ratio of specimen thickness to its radius was 0.4 while the ultimately allowable is considered 0.3. Furthermore, it was assumed that the thermocouple construction described above permits avoiding the influence of temperature fluctuations and makes the self-oscillations more stable. This assumption was confirmed but the thermocouple leads, from foil just as was the thermocouple itself, resulted in additional heat losses. Nevertheless, the main purpose, an experimental verification of the operability of the method, was achieved.

NOTATION

T, temperature; a, thermal diffusivity coefficient; x, coordinate; w, specimen thickness; K, gain coefficient; g, heat-conduction coefficient; S, heater area; R, heater resistance; u_0, reference voltage; a, thermal emf coefficient; x_0, thermocouple coordinate; t, time; σ, Heaviside function; T_{st}, stationary temperature; V, spatial part of the periodic solution; C, a constant; ν, real variable; n, ratio between the thermocouple coordinate and the specimen thickness; ν_j, roots of the characteristic equation; K_0, critical gain coefficient; ε, relative deviation of the gain coefficient from the critical value; δ_j, corrections to the frequency; and $o(\varepsilon)$, a quantity with an order of smallness higher than ε.

LITERATURE CITED

1. A. A. Andronov, A. A. Vitt, and S. É. Khaikin, Theory of Oscillations [in Russian], Moscow (1959).
2. A. M. Letov, Stability of Nonlinear Regulatable Systems [in Russian], Moscow (1962).
3. Yu. S. Kólesov, Differential Equations and Their Application [in Russian], No. 2, 68, Vilnius (1971).
4. V. F. Zavorotnyi and Yu. M. Poplavko, Prib. Tekh. Eksp., No. 5, 189-190 (1984).